Another Approach to Outline Analysis

B - Splines, Control Points

&

MORPHOMATICA

By
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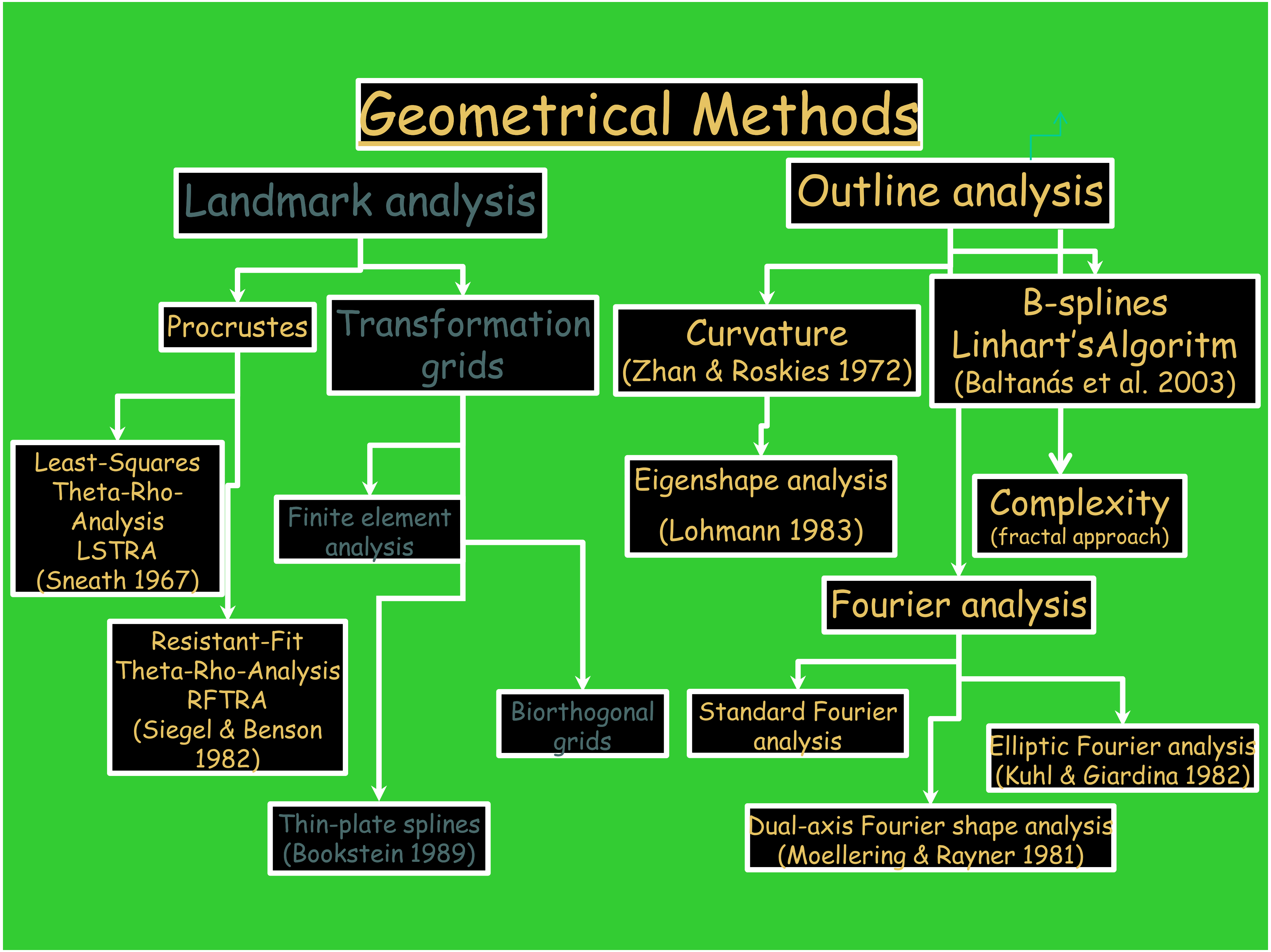
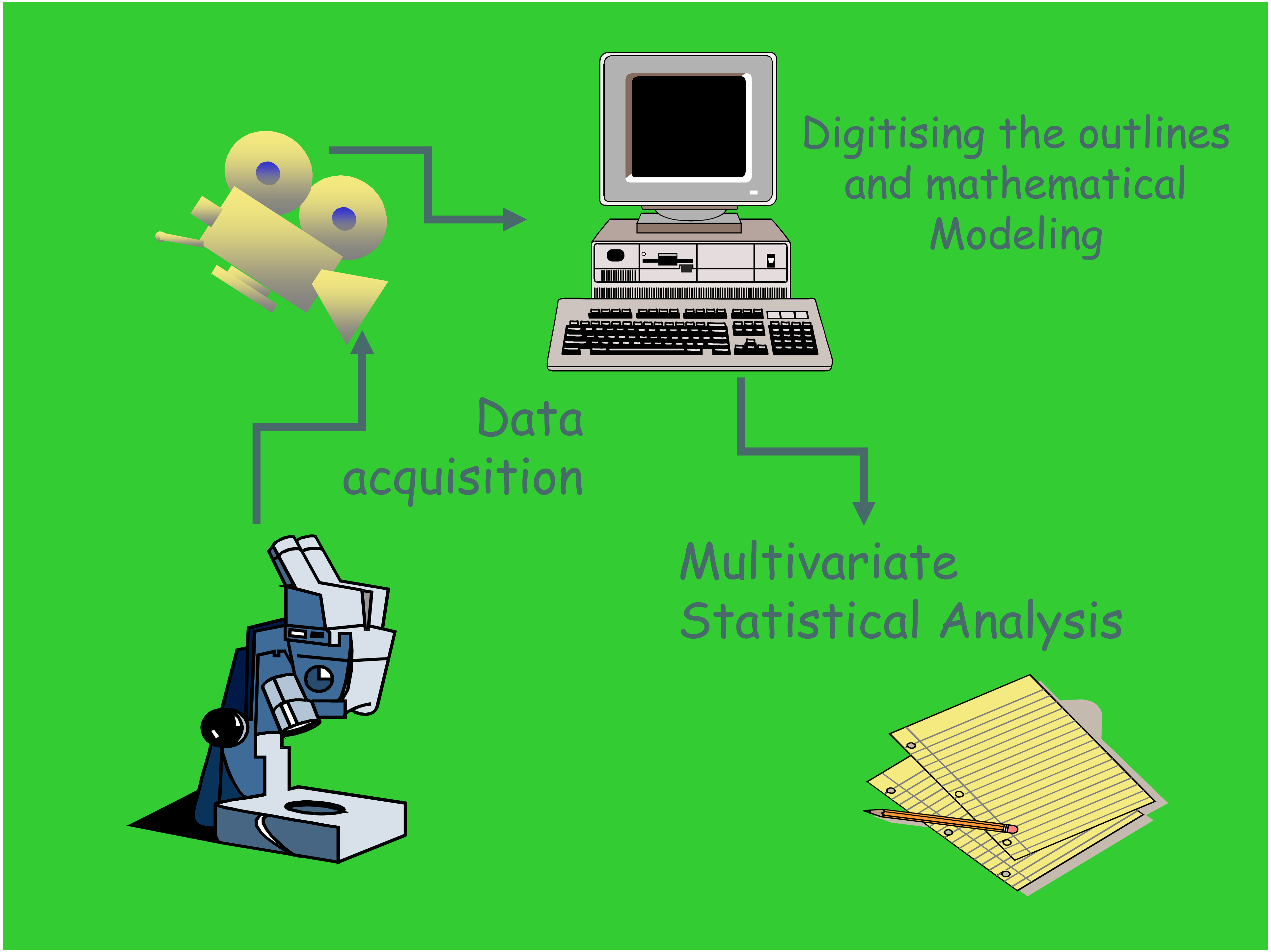
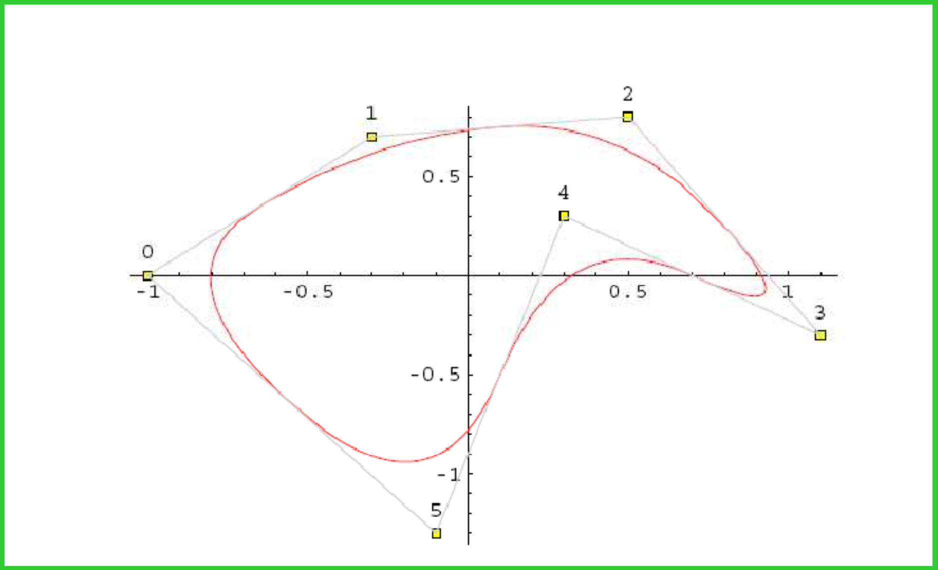
Mag. W. Neubauer, Univ. of Salzburg, Inst. of Mathematics


Geometrical Methods
Landmark analysis
Outline analysis
Procrustes
Transformation grids
Curvature (Zhan & Roskies 1972)
B-splines Linhart'sAlgoritm (Baltanás et al. 2003)
Least-Squares Theta-Rho-Analysis LSTRA (Sneath 1967)
Finite element analysis
Eigenshape analysis (Lohmann 1983)
Complexity (fractal approach)
Fourier analysis
Resistant-Fit Theta-Rho-Analysis RFTRA (Siegel & Benson 1982)
Biorthogonal grids
Standard Fourier analysis
Elliptic Fourier analysis (Kuhl & Giardina 1982)
Thin-plate splines (Bookstein 1989)
Dual-axis Fourier shape analysis (Moellering & Rayner 1981)


Digitising the outlines
and mathematical
Modeling
Data
acquisition
Multivariate
Statistical Analysis

About “Morphomatica”

B-Spline Curves



B-spline curves (the B is an abbreviation of basis) were developed by Carl de Boor due to his researches at General Motors. They allow to describe virtually any curve by a few „control points“ up to a certain tolerance.

Nowadays these curves are a really well-established tool in computer-aided design and computer graphics. They are used in

•car design or ship construction

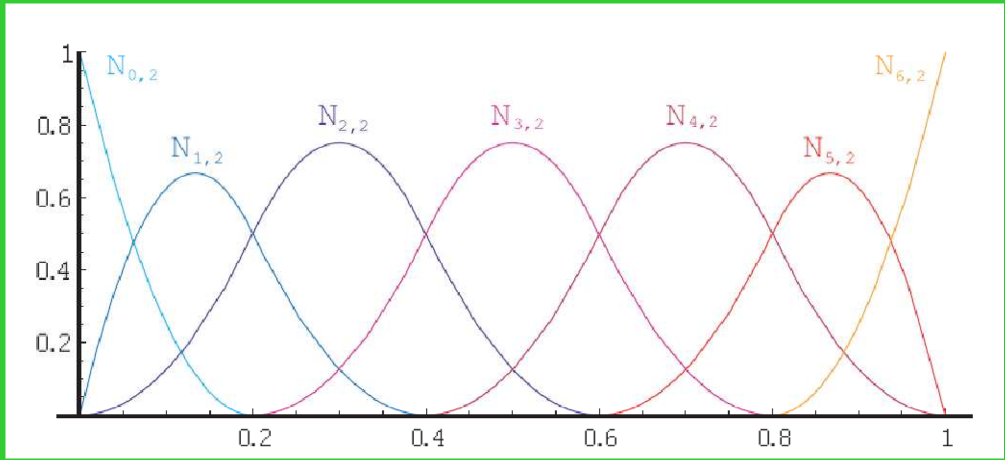
•postscript-converters

•laying of rail trays and construction of roller coasters

•in many other applications in which an exact description of free form curves is necessary.

<u>B-Spline Curves</u>

B-spline curves are parametric curves defined piecewise by polynomials of a specified degree *p*, called <u>basis functions</u>. They are iteratively defined by



Basis functions of degree 2 using a knot vector with equidistant knot values.

$$N_{i,0}(t) = \begin{cases} 1 & \text{if } u_i \le t < u_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$$N_{i,p}(t) = \frac{t - u_i}{u_{i+p} - u_i} N_{i,p-1}(t) + \frac{u_{i+p+1} - t}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(t),$$

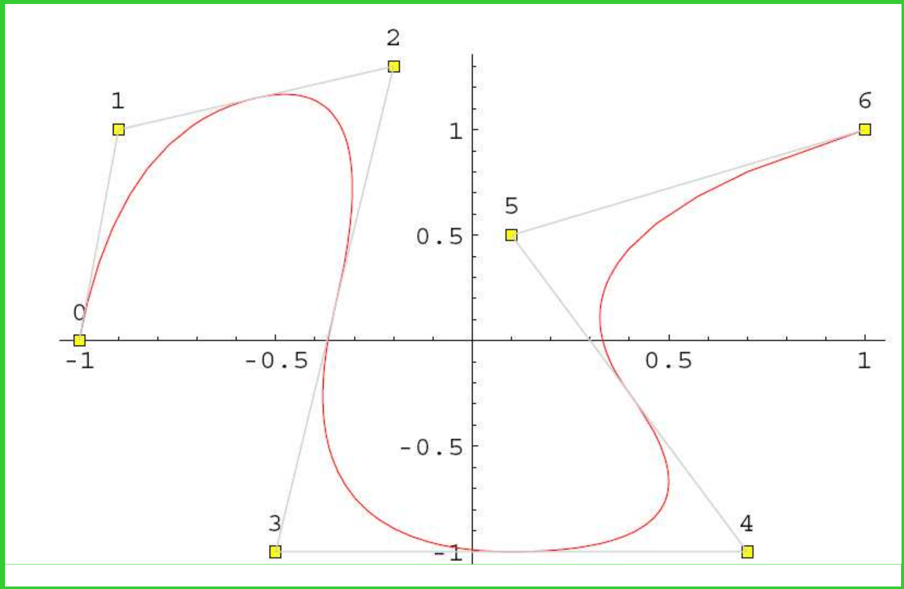
where $U = (u_0,\ldots,u_m)$ is a nondecreasing sequence of real numbers, i.e., $u_i \le u_{i+1}$, $i = 0,\ldots,m-1$, called <u>knot vector</u>.

B-Spline Curves

We obtain a B-spline curve **C**(t) of degree p easily by

$$\mathbf{C}(t) = \sum_{i=0}^{n} N_{i,p}(t)\,\mathbf{P}_i \qquad a \le t \le b.$$

where the $\mathbf{P}_i$ are the control points, and the $N_{i,p}(t)$ are the pth-degree B-spline basis functions. The polygon formed by the $\mathbf{P}_i$ is the control polygon.



B-spline curve of degree 2 formed by the control points $\{\mathbf{P}_i\}=\{(-1,0),(-0.9,1),(-0.2,1.3),(-0.5,-1),(0.7,-1),(0.1,0.5),(1,1)\}$.

<u>B-Spline Curves</u>

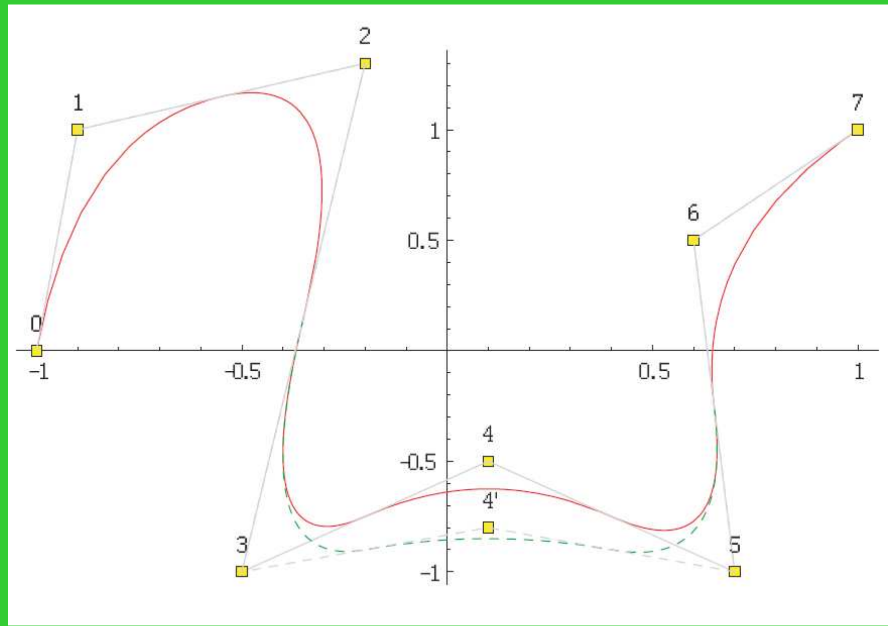
Properties:

•The B-spline curve is determined by its control points. We can use these points as <u>Pseudo-Landmarks</u>.

•The control polygon represents a kind of approximation to the curve.

•<u>Local support</u>: Moving a single control point alters the curve just on the part close to the adjusted control point

•<u>Affine invariance</u>: An affine transformation, including translation, rotation and scaling, is applied to the curve by applying it to the control points.



Local support: Moving $\mathbf{P}_4$ to $\mathbf{P}_{4'}$ changes the curve‘s progression just in the parameter interval [2/6,5/6].

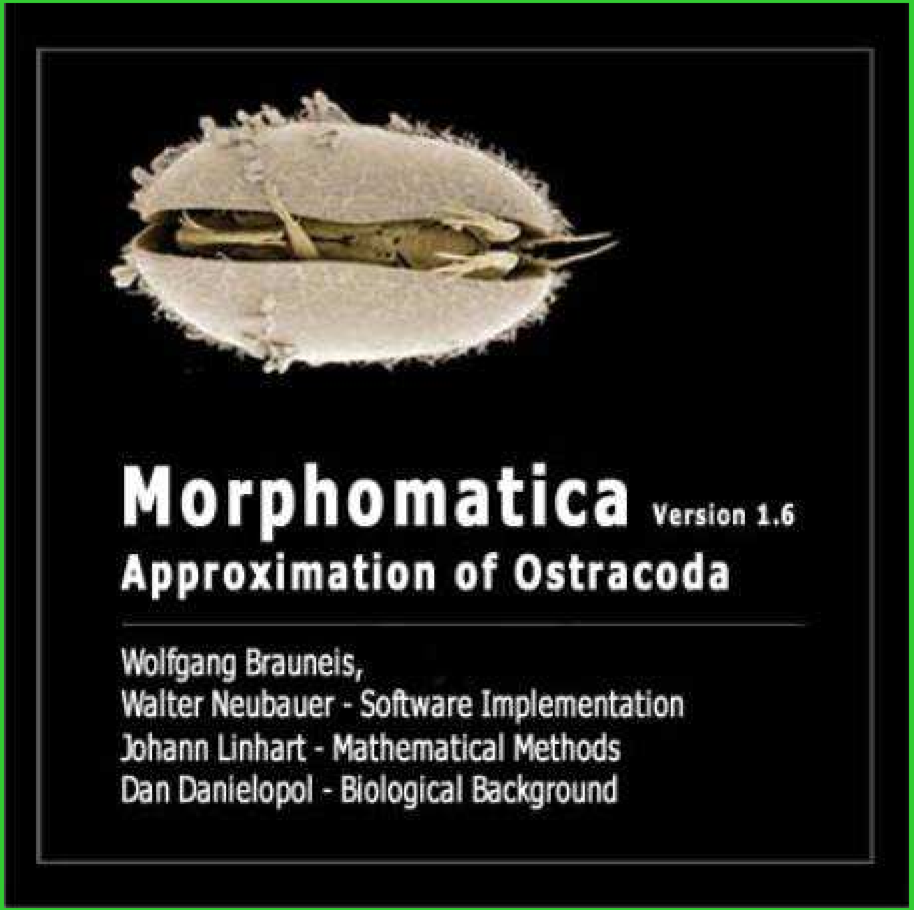
Morphomatica (current version 1.6) uses B-spline curves to approximate the pixel data of ostracod outlines.

The program, evolved under the leadership of Dan L. Danielopol, was written by Wolfgang Brauneis during 2001 - 2007.

Johann Linhart adapted the B-splines to approximate ostracode outlines.

Download at :

http://palstrat.uni-graz.at

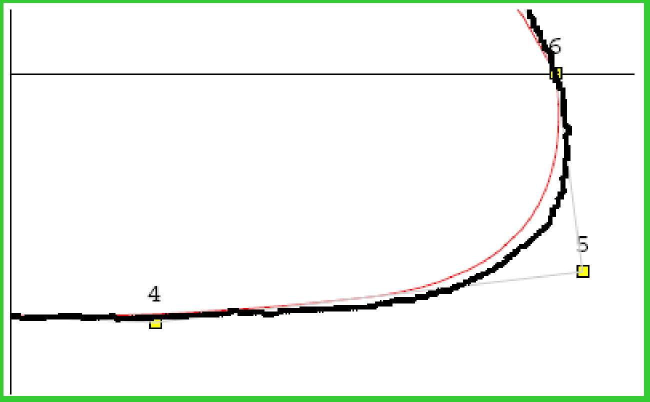


Approximation of the outline data

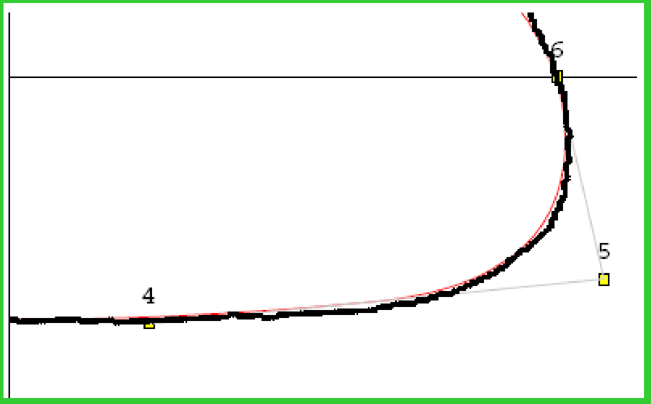
Tps-files usually contain 1000 – 1500 coordinates of pixel. We want to approximate them by usually 16 control points.

Linhart's algorithm (using a singular value decomposition and a pseudo-inverse matrix to solve the overdetermined system of equations) provides a good and numerically stable approximation to the outline data.

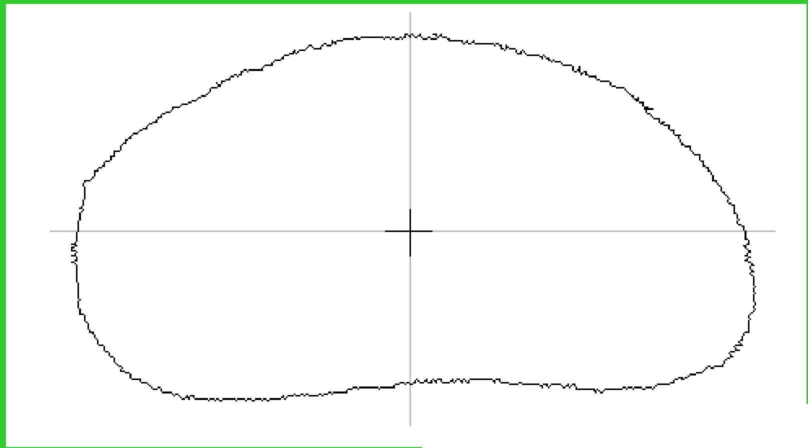
Morphomatica offers an iterative procedure called „parameter correction", to get a better fitting B-spline curve.

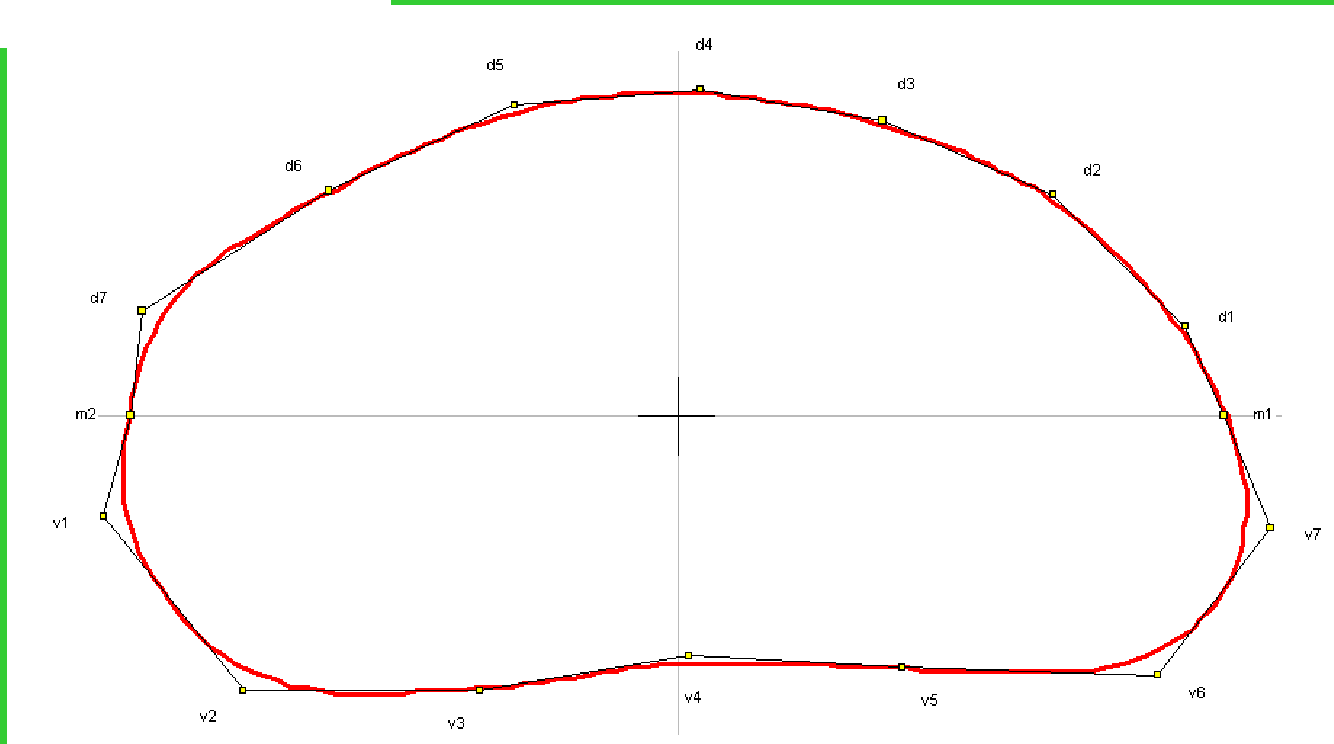


Before a parameter correction.



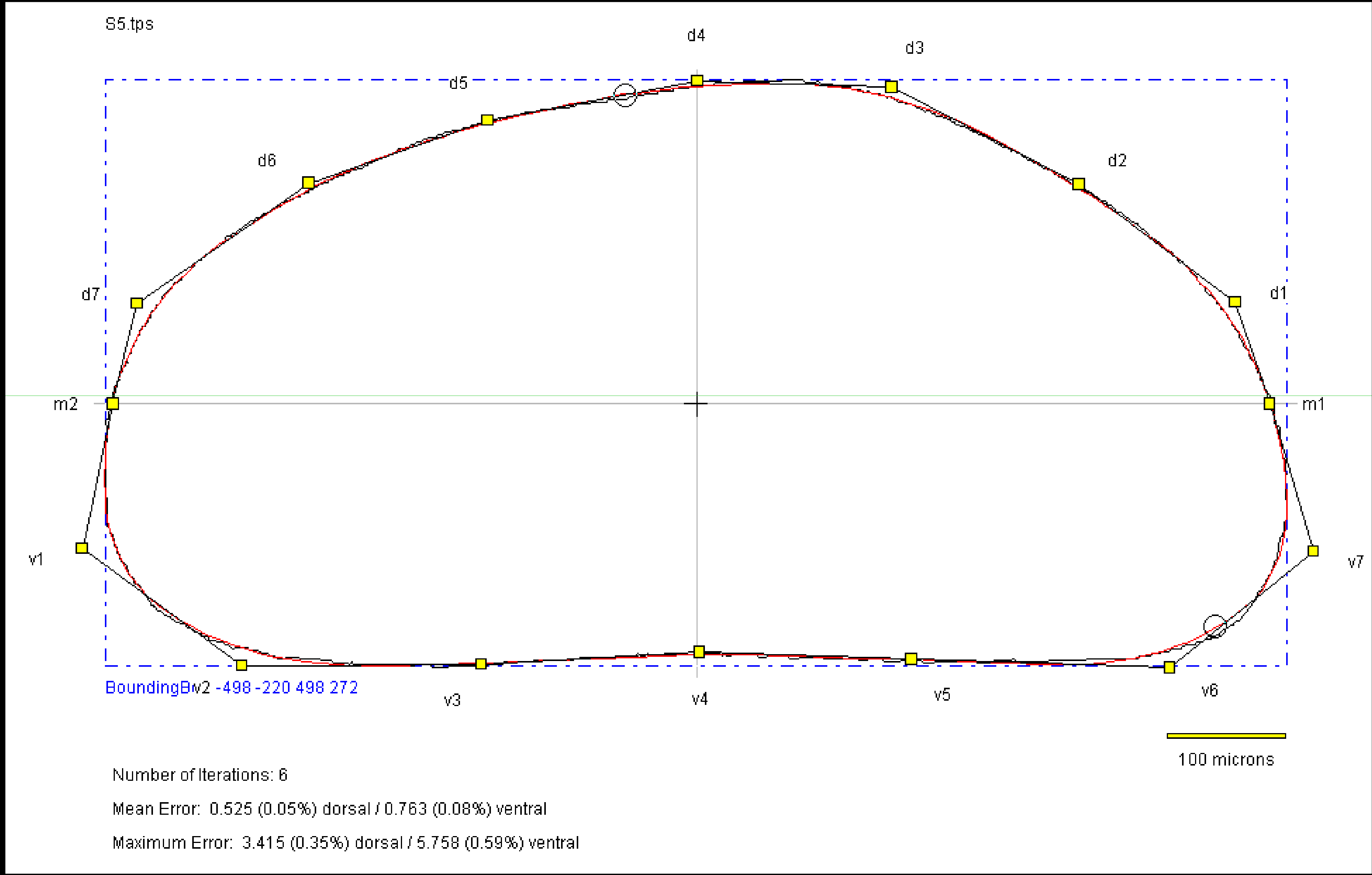
After 6 iterative steps a better approach is guaranteed.





B-spline curves provide an excellent method to characterize an outline by a small number of parameters, namely the coordinates of the control points.

Linhart's algorithm



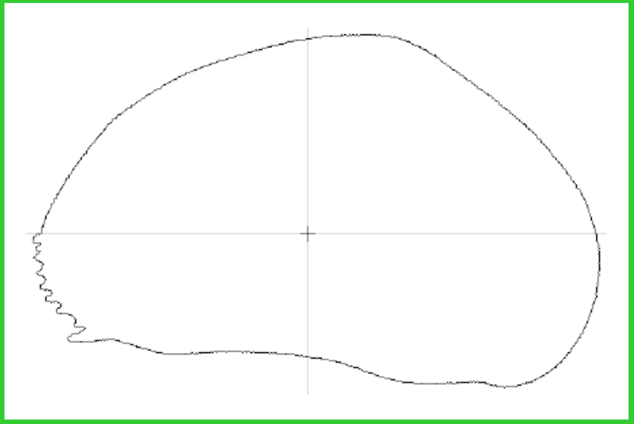
Comparing B-spline curves

For a reasonable comparison of the approximating B-spline curves a transformation into a superimposed position is essential. This can easily be done by an affine transformation consisting of a translation, a rotation, and a scaling.

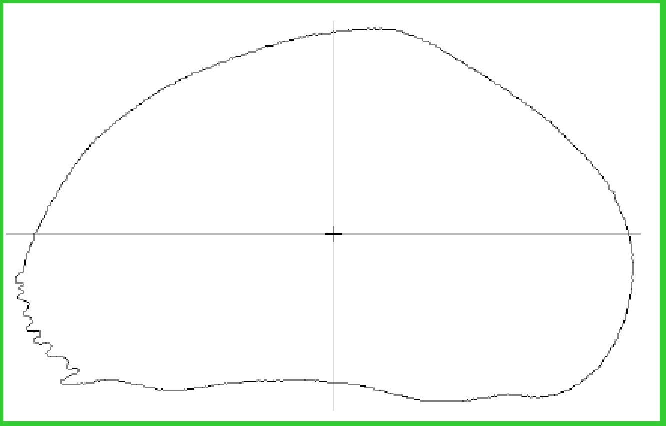
1. The centre of gravity of the domain surrounded by the curve is in the origin.

2. The axis of minimum moment of inertia corresponds with the *x*-axis.

It is essential to use the centre of gravity and the axis of minimum moment of the surrounded domain! Computing the centroid and the axis of minimum moment of inertia using the pixel data leads to an insufficient result if the outlines have rugged or jagged parts.



Centroid and main axes computed with the outline points.

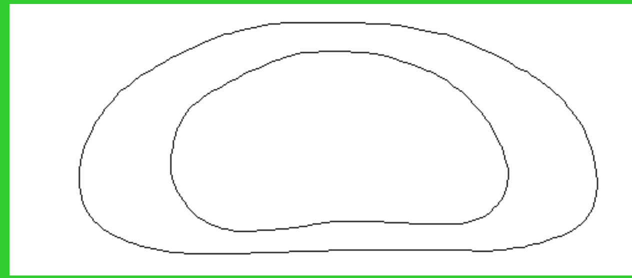


The origin now is the centre of gravity of the domain and the axes correspond with the main axes of inertia.

Scaling

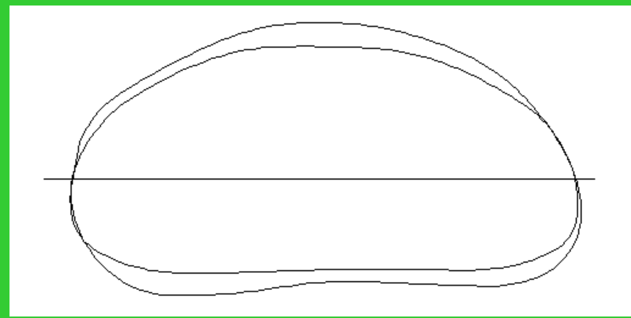
- No normalization

  The size of the outlines remain unaffected.



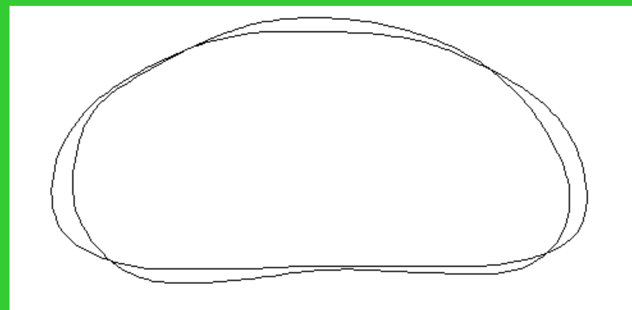
- Normalize for outer control points

  The points of the outline crossing the x-axis are set (-1000,0) resp. (1000,0).



- Normalize for area

  The area of the surrounded domain of both curves is equal.

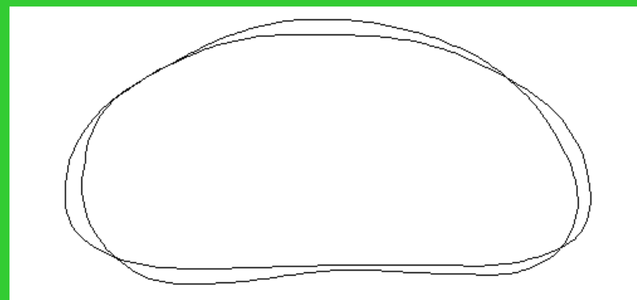


- Normalize for centroid size

  The centroid size, given by

$$\sqrt{\frac{1}{k}\sum_{i=1}^{k}\left((x_i-\overline{x})^2+(y_i-\overline{y})^2\right)}$$

  of both curves is equal.

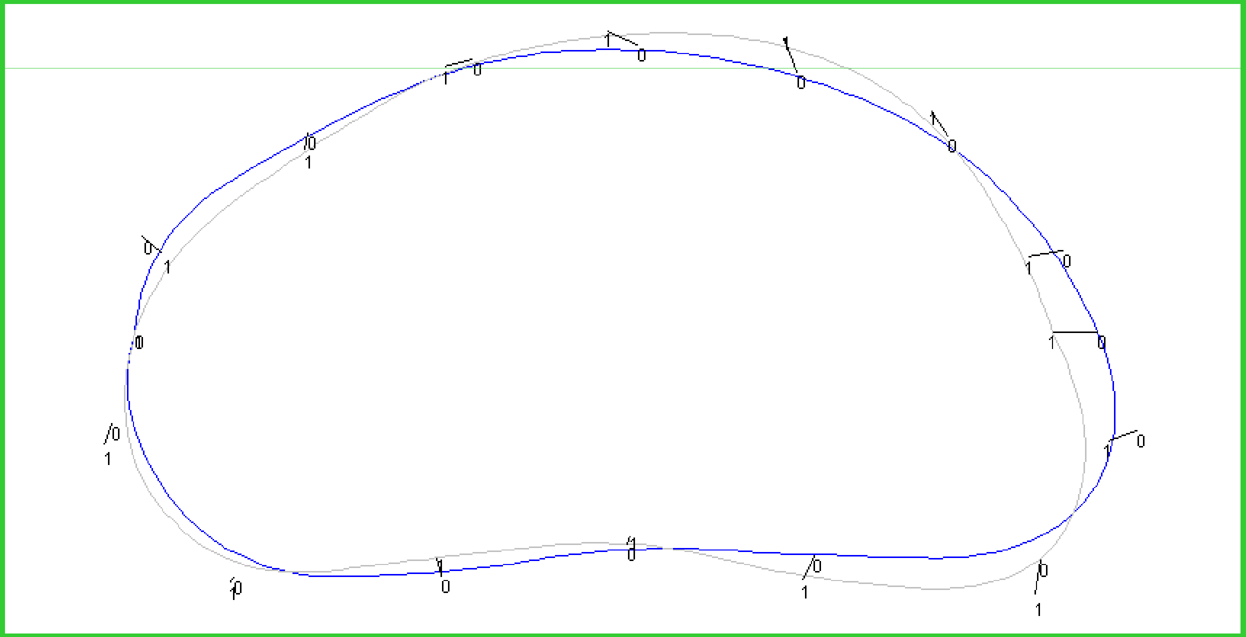


Measure 1: Mean delta square

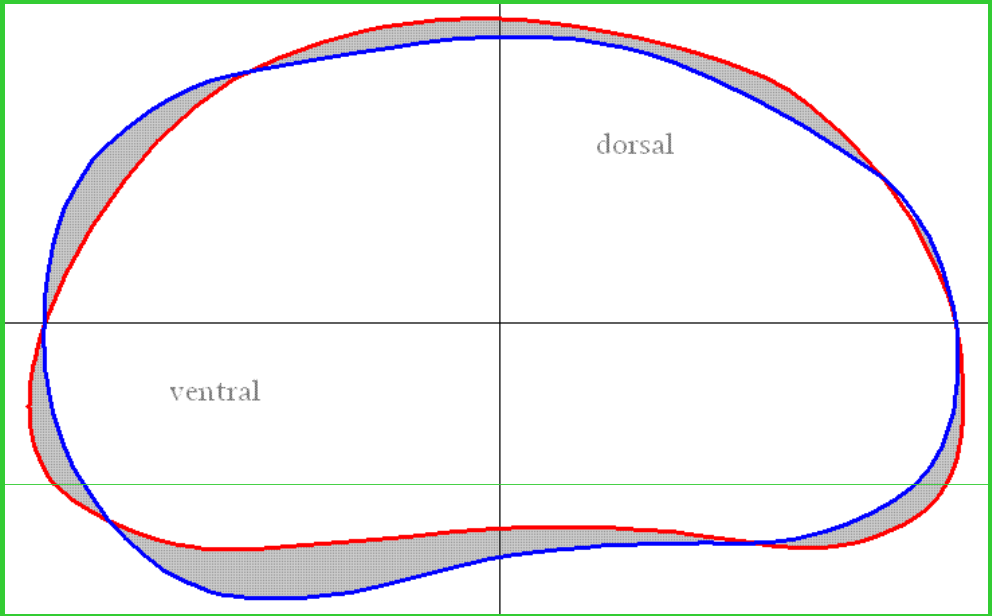
Mean delta square uses the control point as pseudo-landmarks.

We define the difference *d* of two approximating B-spline curves **C** and **D** with control point sequences $\mathbf{P}_1,\ldots,\mathbf{P}_n$ resp. $\mathbf{Q}_1,\ldots,\mathbf{Q}_n$ as

$$d\left(\mathbf{C},\mathbf{D}\right) := \frac{1}{n}\sqrt{\sum_{i=0}^{n} \|\mathbf{P}_i - \mathbf{Q}_i\|^2}.$$



Measure 2: Area Deviation



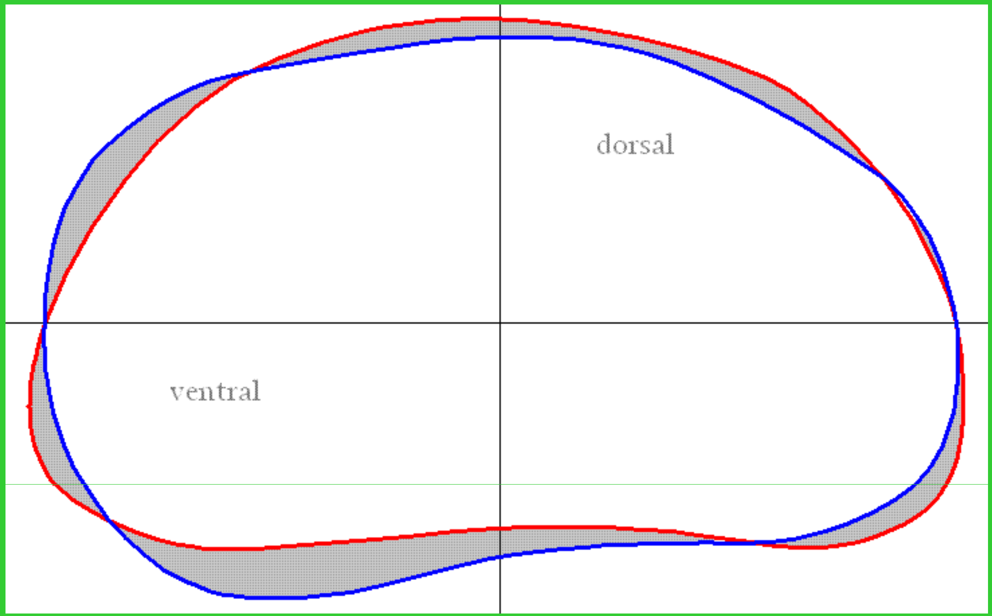
The light grey area is the area deviation of two superimposed B-spline curves.

The area deviation is the area of the part of the plane that is contained in the interior of exactly one of the two outlines.

It may be seen as the area „between“ the curves.

Measure 2: Area Deviation

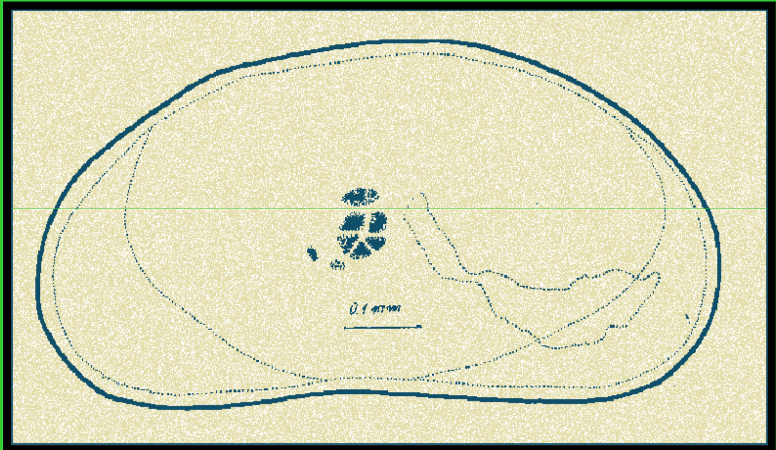
- Common and natural distinction measure
- Demonstrative and tangible
- Resulting differences in µm²
- Inured to possible data errors and measuring faults
- Ostracode outlines feature good characteristics for a fast computation



Morphomatica is able to compute the area deviation of the whole outline or restricted to the dorsal resp. ventral region only.

OUTLINE ANALYSIS OF Cryptocandona

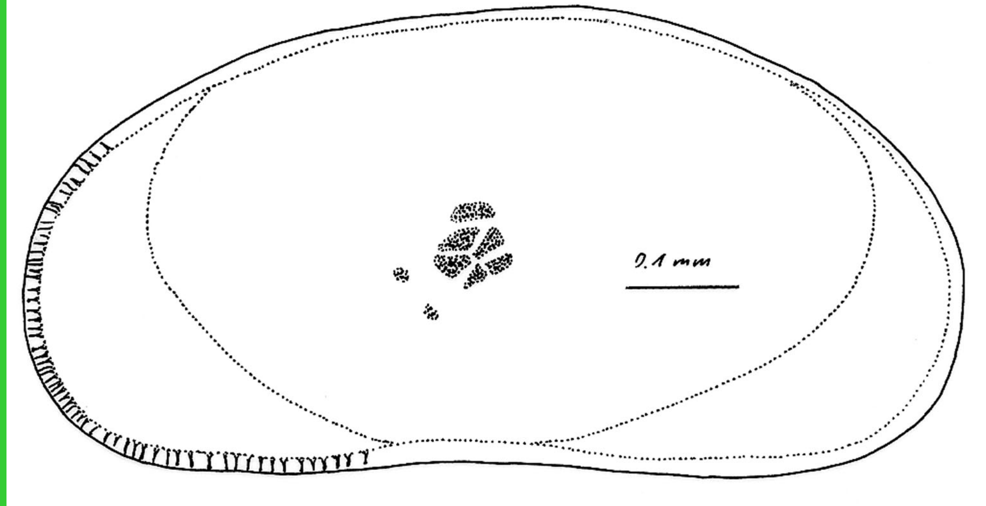
The genus *Cryptocandona* Kaufmann 1900 represents a primitive phylogenetic lineage within the SF. Candoninae (cf. thoracic limbs & hemipenis in Namiotko et al., 2005a, 2005b)

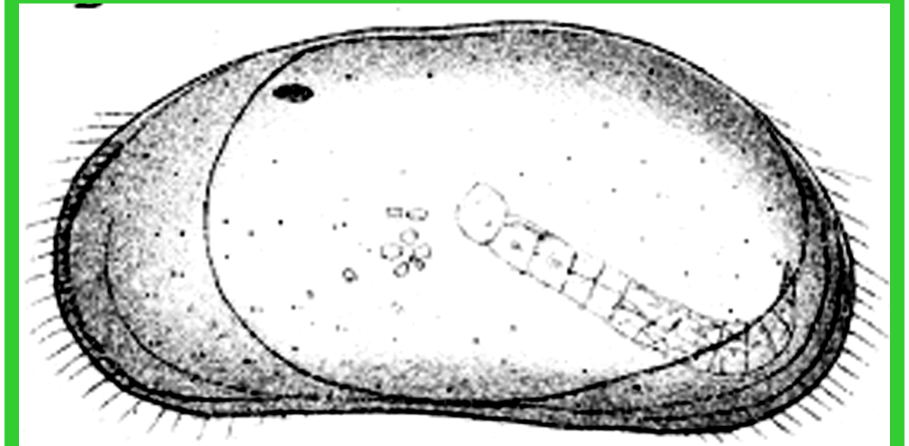


Cryptocandona vavrai **Kaufmann 1900 is a species widely spread in Europe, especially in running waters like springs and in superficial groundwater (Löffler & Danielopol, 1978). The species is easily recognisable using the valve & the limbs traits. The species reproduces parthenogenetically and seldom males were collected & studied (cf. Namiotko et al., 2005b).**

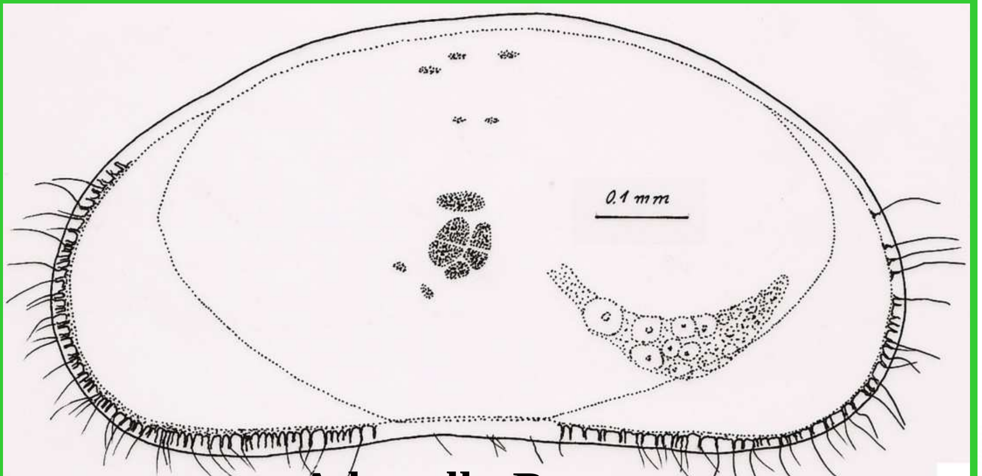
Additional material helping to reconstruct the evolutionary cronicle of *Cr. vavrai* lineage

La Clou de la fou

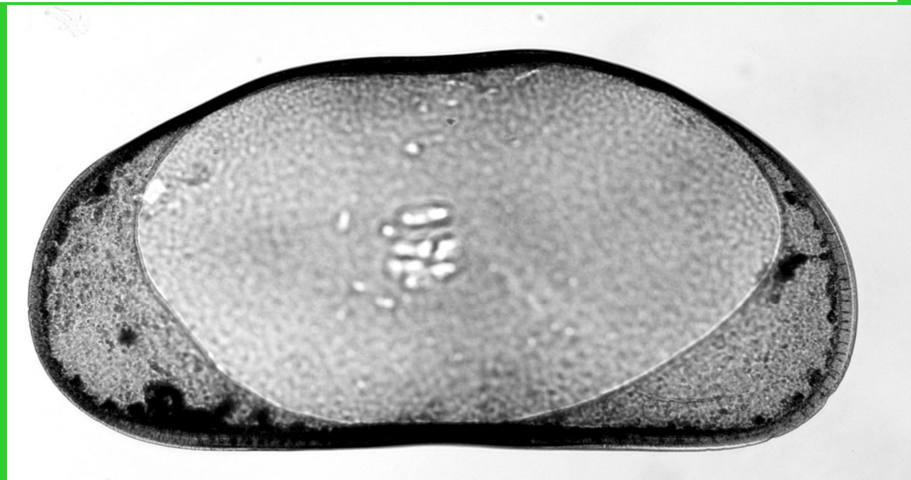




Gentilino –locality type for Cr. vavrai



Adamello-Brenta



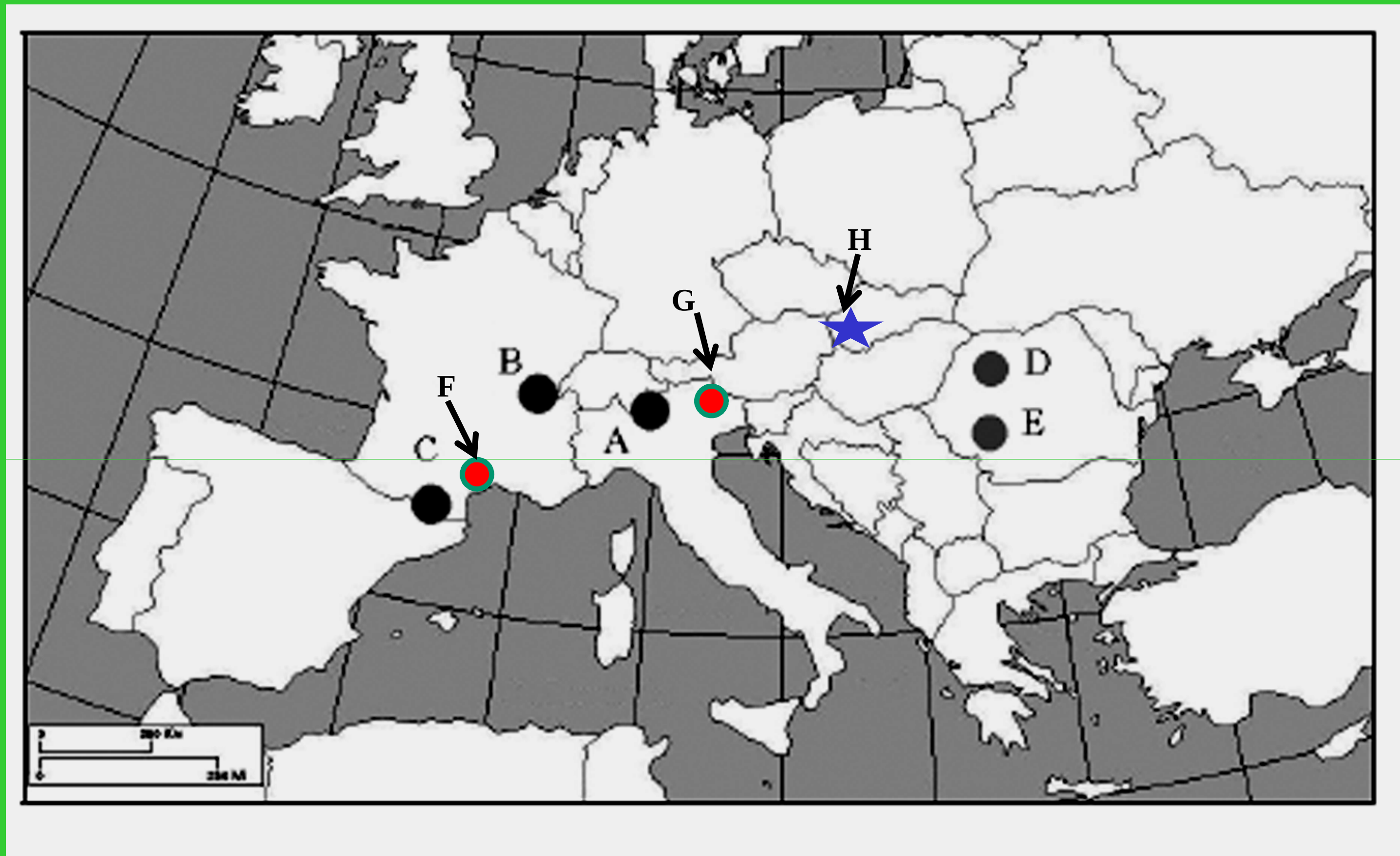
Slovakia - PID

200 µm

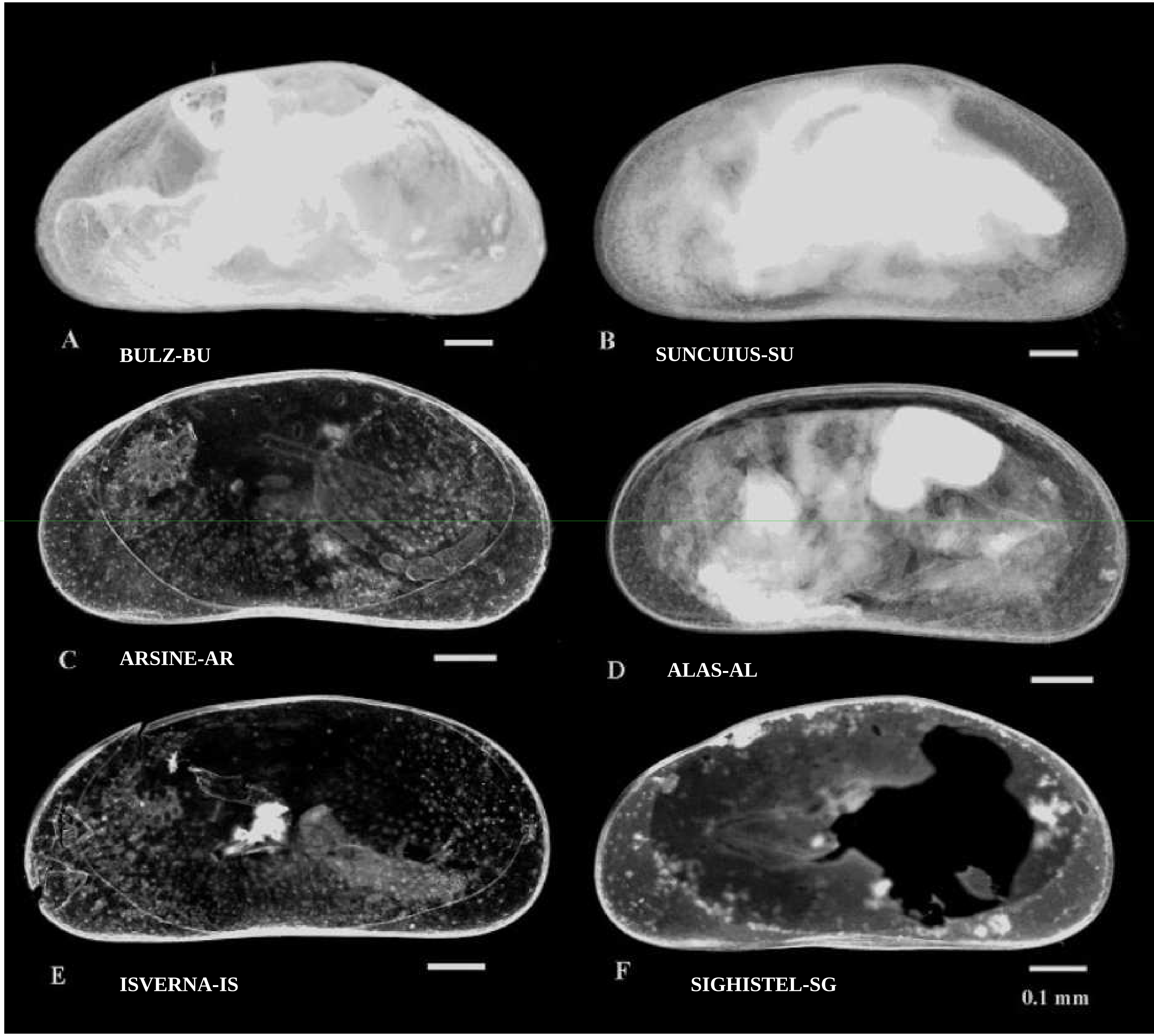
Biometric data

		AL	AR	BU	SU	SG	IS
LM (mm)	Mean	0.86	0.83	1.03	1.02	0.95	0.92
	SD	0.03	0.03	0.04	0.01	0.00	0.06
	Range	0.80-0.90	0.77-0.87	0.97-1.09	0.99-1.02	0.95	0.85-0.95
HM (mm)	Mean	0.41	0.41	0.50	0.50	0.47	0.44
	SD	0.01	0.01	0.03	0.009	0.00	0.04
	Range	0.39-0.42	0.38-0.42	0.47-0.56	0.48-0.51	0.47	0.38-0.45
HM/LM (%)	Mean	48	49	49	48	49	46
	SD	1	1	2	1	0	1
	Range	46-50	47-51	47-53	48-50	49	45-47

Main areas where *Cr. vavrai* were sampled for this study (cf. Baltanás et al., 2000, Iepure et al., 2007, MS submitted)



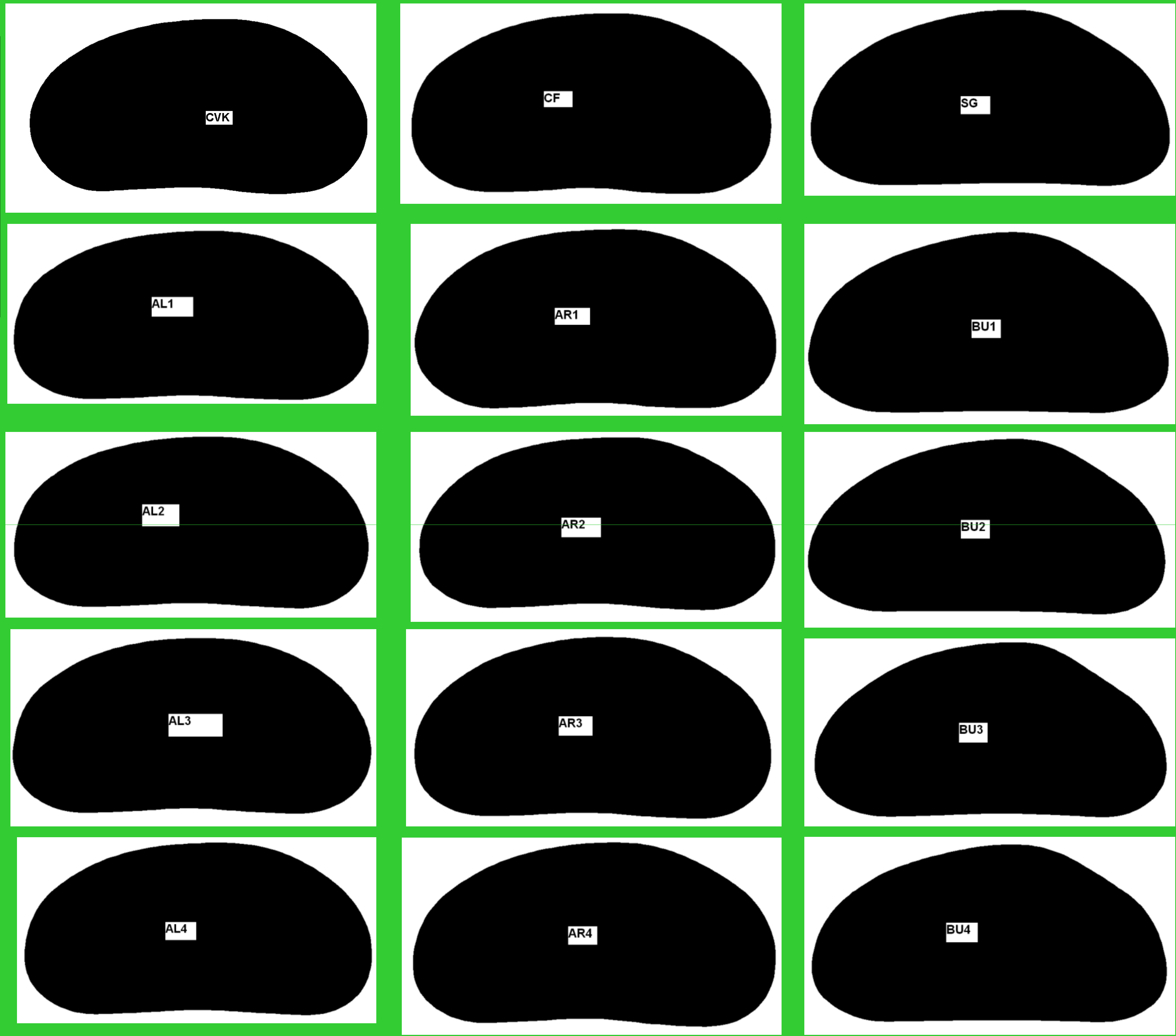
A-Gentilino (Kaufmann 1900); B- Arsine; C-Alas; D-Suncuius, Bulz & Sighistel; E-Isverna; F- La Clou de la fou; G-Adamello-Brenta; H-PID (Miocene-fossil).


A
BULZ-BU
B
SUNCUIUS-SU
C
ARSINE-AR
D
ALAS-AL
E
ISVERNA-IS
F
SIGHISTEL-SG
0.1 mm

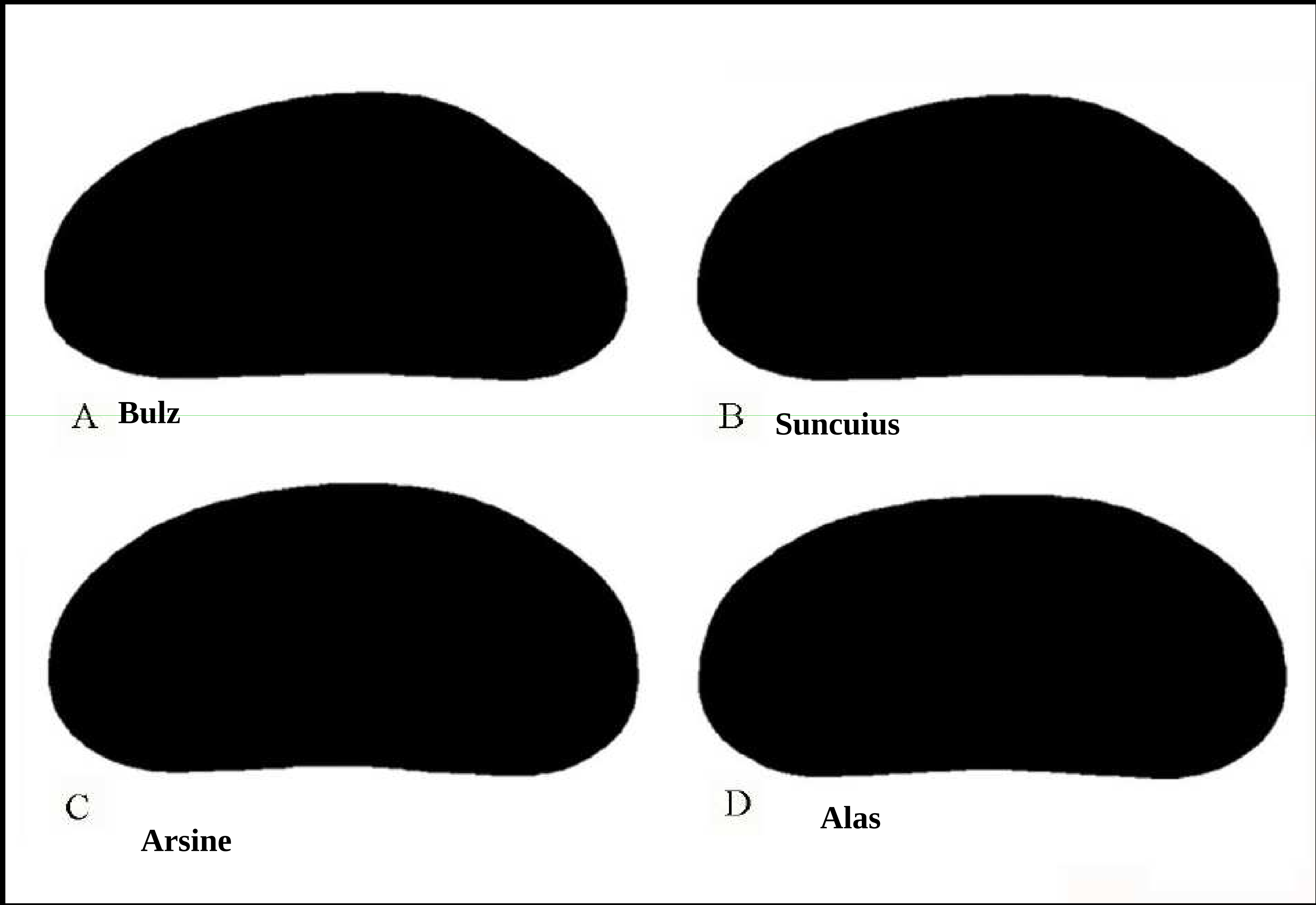
Shape Analysis

Variability left valves:

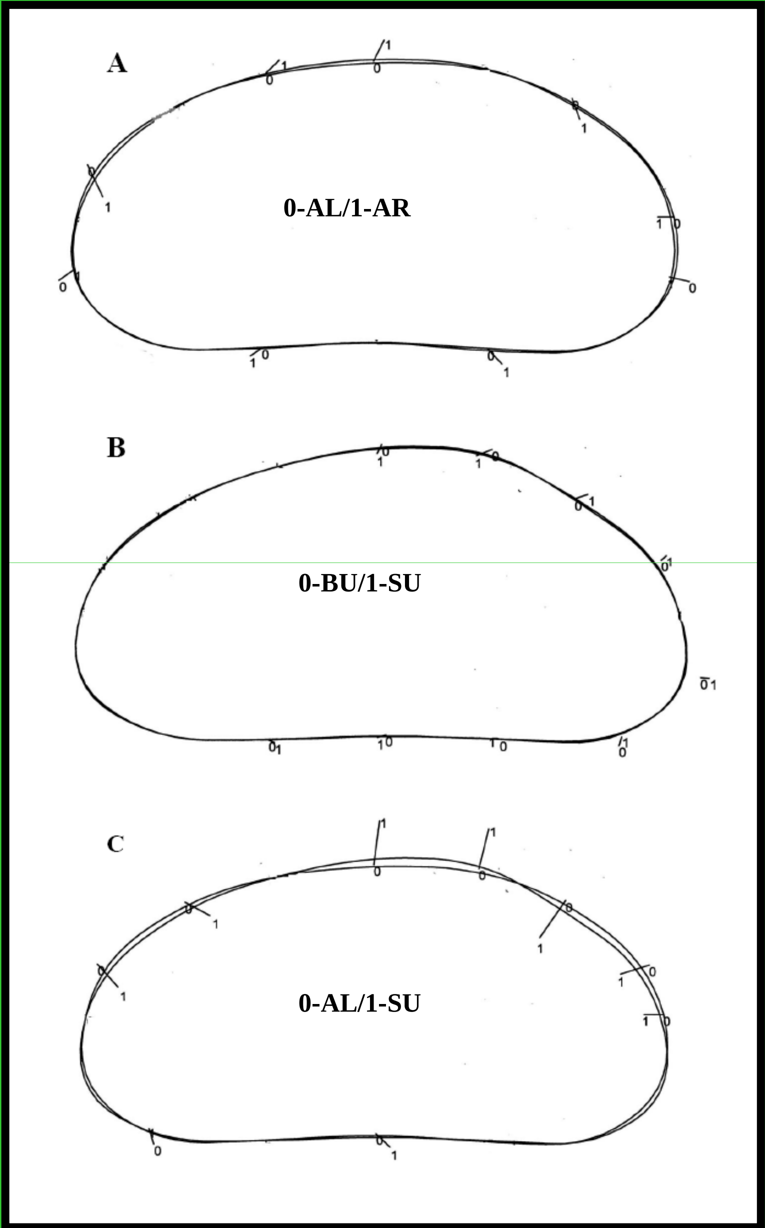
CVK-Gentilino, CF-La Clou d.Fou, SG-Sighistel, AL-Alas, AR-Arsine, BU-Bulz

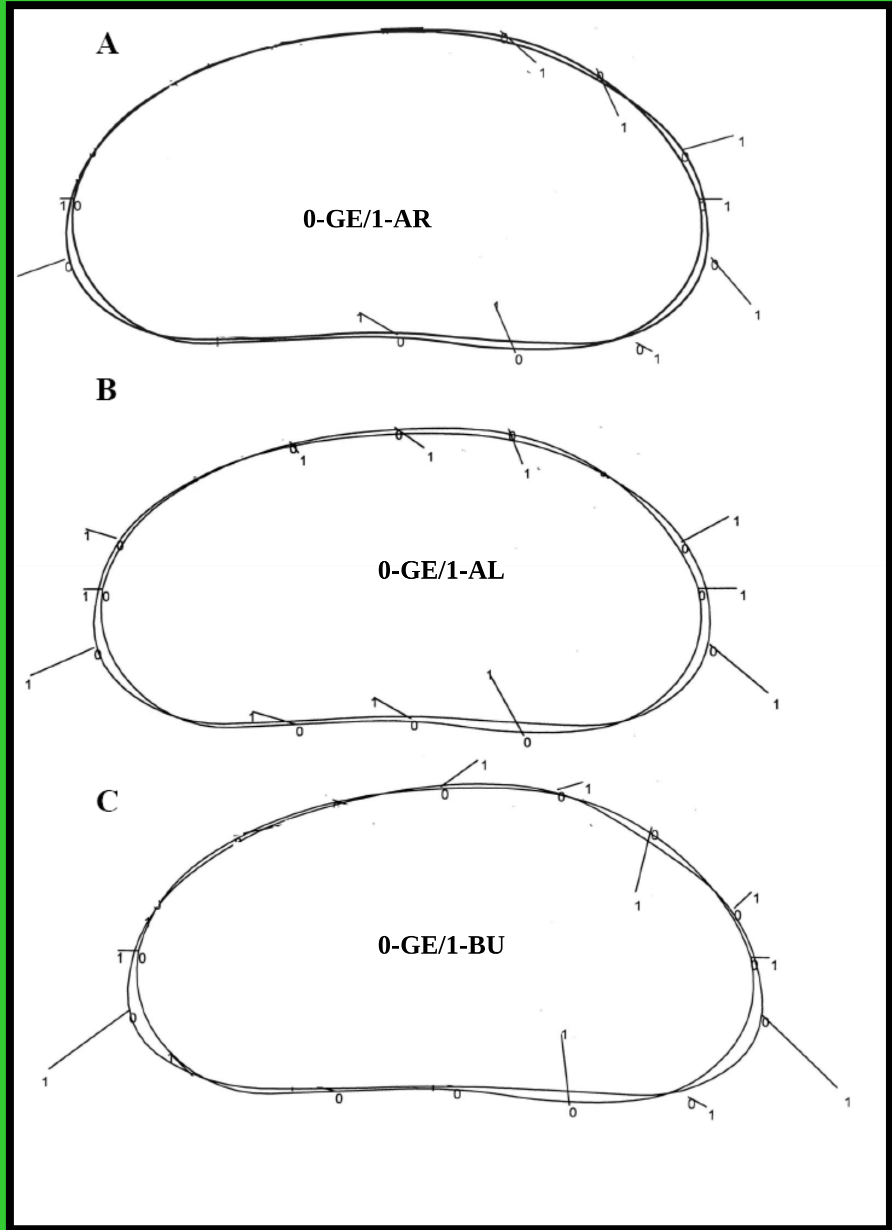


Virtual mean outlines for left valves (n = 8/ population)

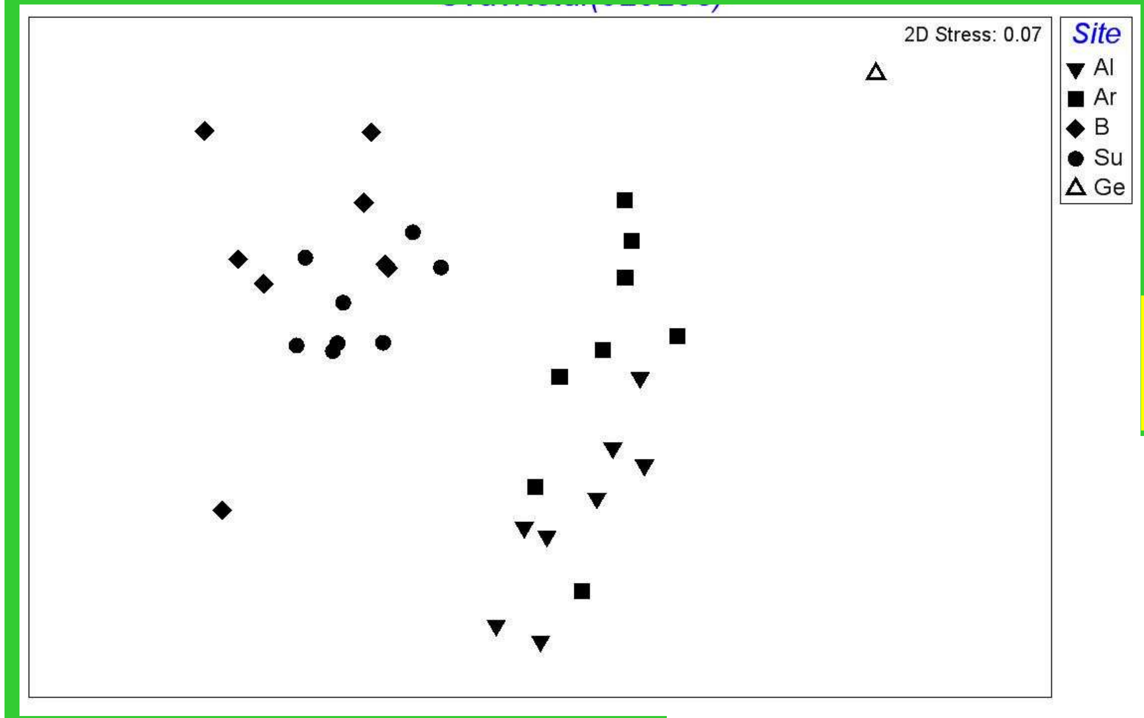


Superimposition of mean shapes

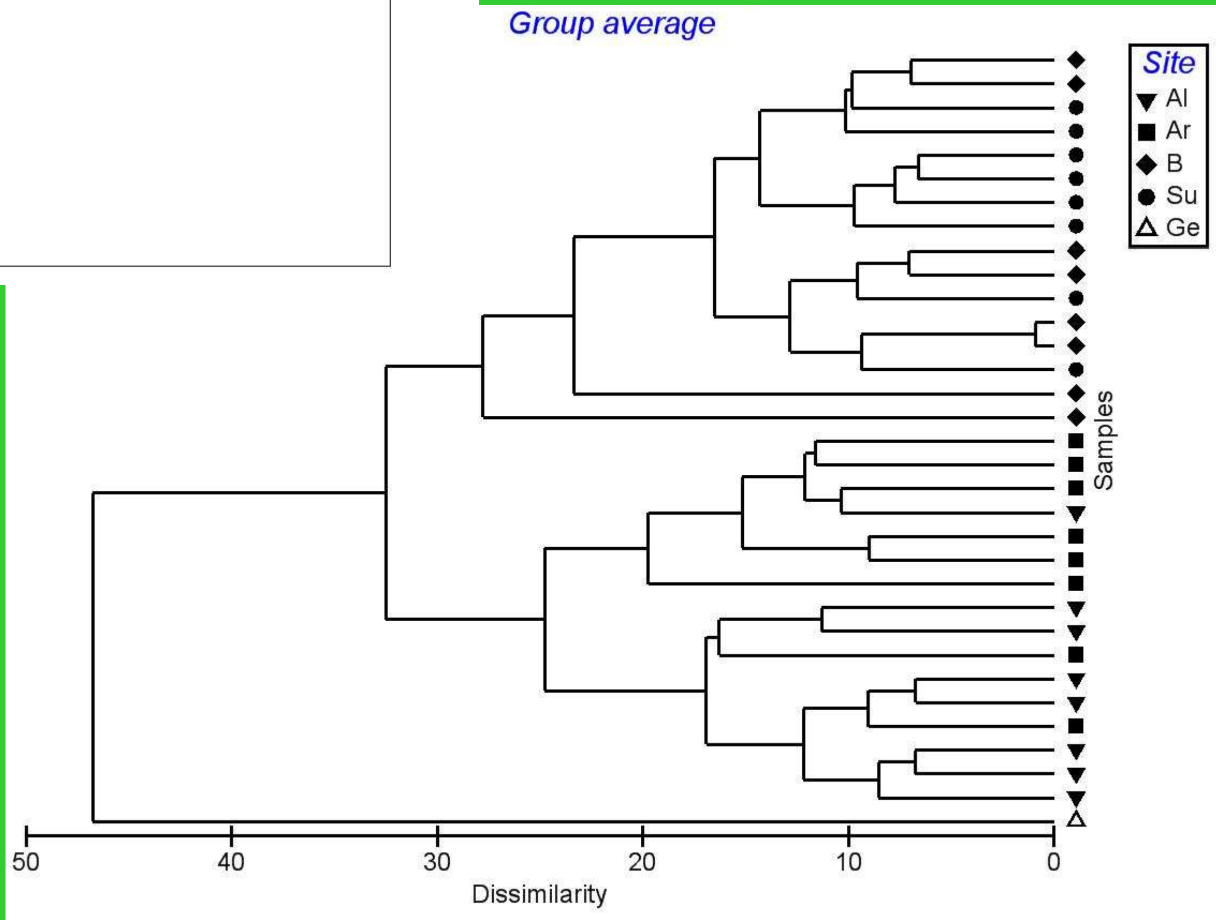




Multi Dimensional Scaling-MDS (non-metric!), cf. Iepure et al. 2008.



UPGMA-Cluster based on valve disparities



$$\text{MPD} = \sum \text{Disparities} / n(n-1)/2$$

	MPD	CL95%
AL	14.90	12.69-17.25
AR	20.45	17.90-23.00
BU	20.26	16.80-23.74
SU	12.57	11.36-13.79

An example of matrix for computation of the Linhart's Disparity Index

Arcine (France) –
Adamello (Italy)

	Ar1	Ar2	Ar3	Ar4	Ar5	Ar6	Ar7	Ar8	It1	It2	It3	It4	It5
Ar1													
Ar2	27.03												
Ar3	19.13	22.04											
Ar4	33.1	14.87	27.66										
Ar5	33.17	17.39	22.97	9.02									
Ar6	28.08	13.94	18.93	13.14	11.24								
Ar7	35.3	20.45	27.55	18.9	21.31	19.43							
Ar8	24.07	11.84	13.42	16.48	16.24	11.61	16.34						
It1	64.58	42.48	61.5	34.31	39.41	45.13	44.07	49.74					
It2	53.68	30.34	49.77	22.2	28.31	33.75	29.8	37.39	18.75				
It3	83.19	60.2	81	53.84	60.11	65.69	58.03	68.39	29.65	32.54			
It4	66.24	44.24	63.86	36.4	42.25	48.05	41.59	51.32	18.39	18.8	22.5		
It5	67.94	46.48	66.81	40.38	47.32	52.33	47.06	54.68	25.64	21.45	20.68	20.65	

WD_1

BD

WD_2

LDI = 1.02

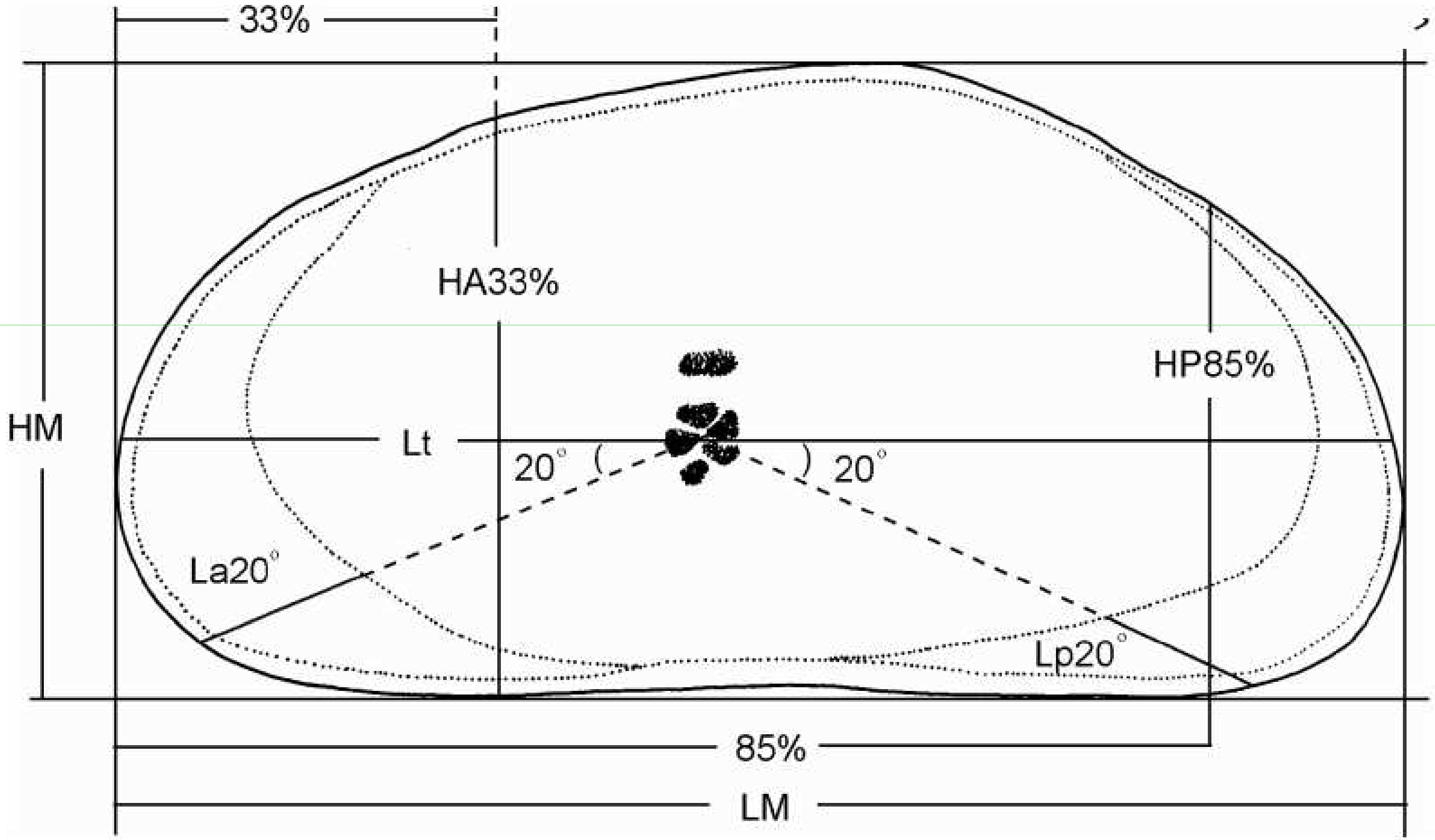
Results of the one way ANOSIM test for the differences (expressed by the R statistic values with the probability levels) in the total, dorsal and ventral area of the female valves between pairs of the studied populations.

R= (Mean rB – Mean rW)/0.5(n(n-1)/2)
r-rang of pairwise disparities , B-between populations, W-within population.

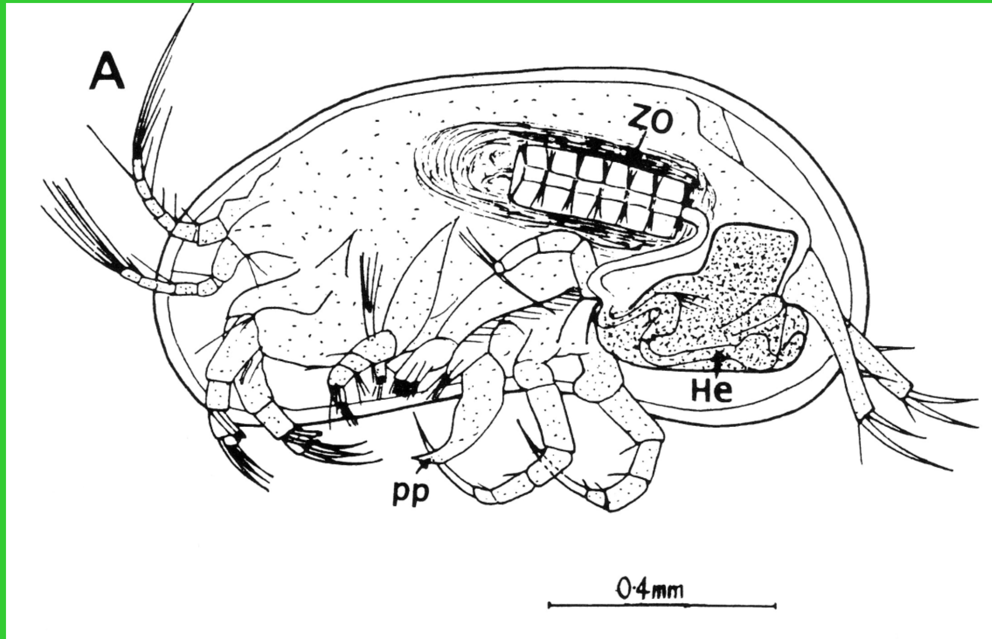
Populations	Total area		Dorsal area		Ventral area	
	R statistic	P level (%)	R statistic	P level (%)	R statistic	P level (%)
AR - BU	0.80	0.1	0.68	0.2	0.68	0.1
AR - SU	0.81	0.1	0.66	0.1	0.82	0.1
AR - AL	0.31	0.9	0.37	0.5	0.10	9.3
BU - SU	0.16	3.4	0.22	1.2	0.02	30.3
BU - AL	0.92	0.3	0.94	0.1	0.59	0.1
SU - AL	0.98	0.1	0.99	0.3	0.77	0.1

Description of the ANOSIM algorithm, in Clarke & Warwick, 2001

Protocol for measurements of distances between landmarks/pseudolandmarks



Theoretical morphology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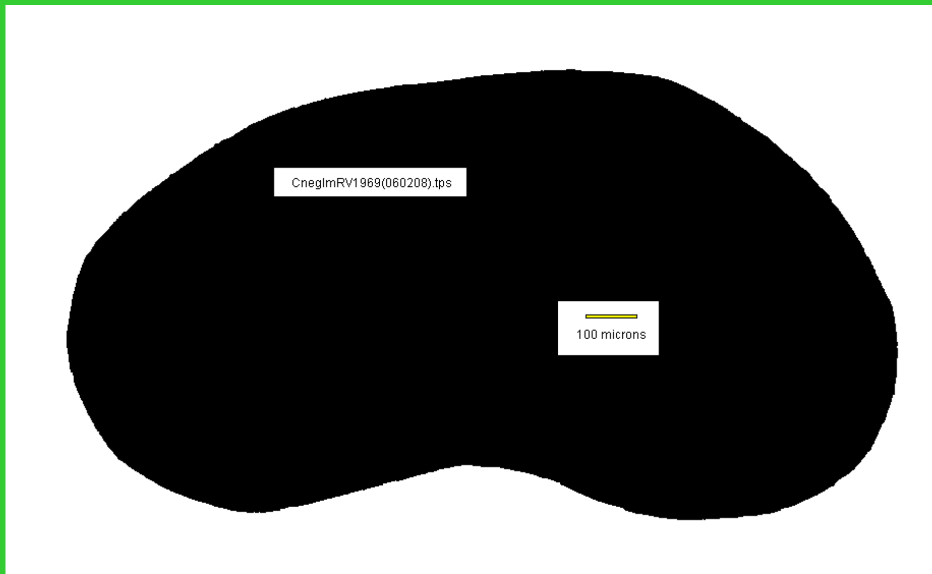
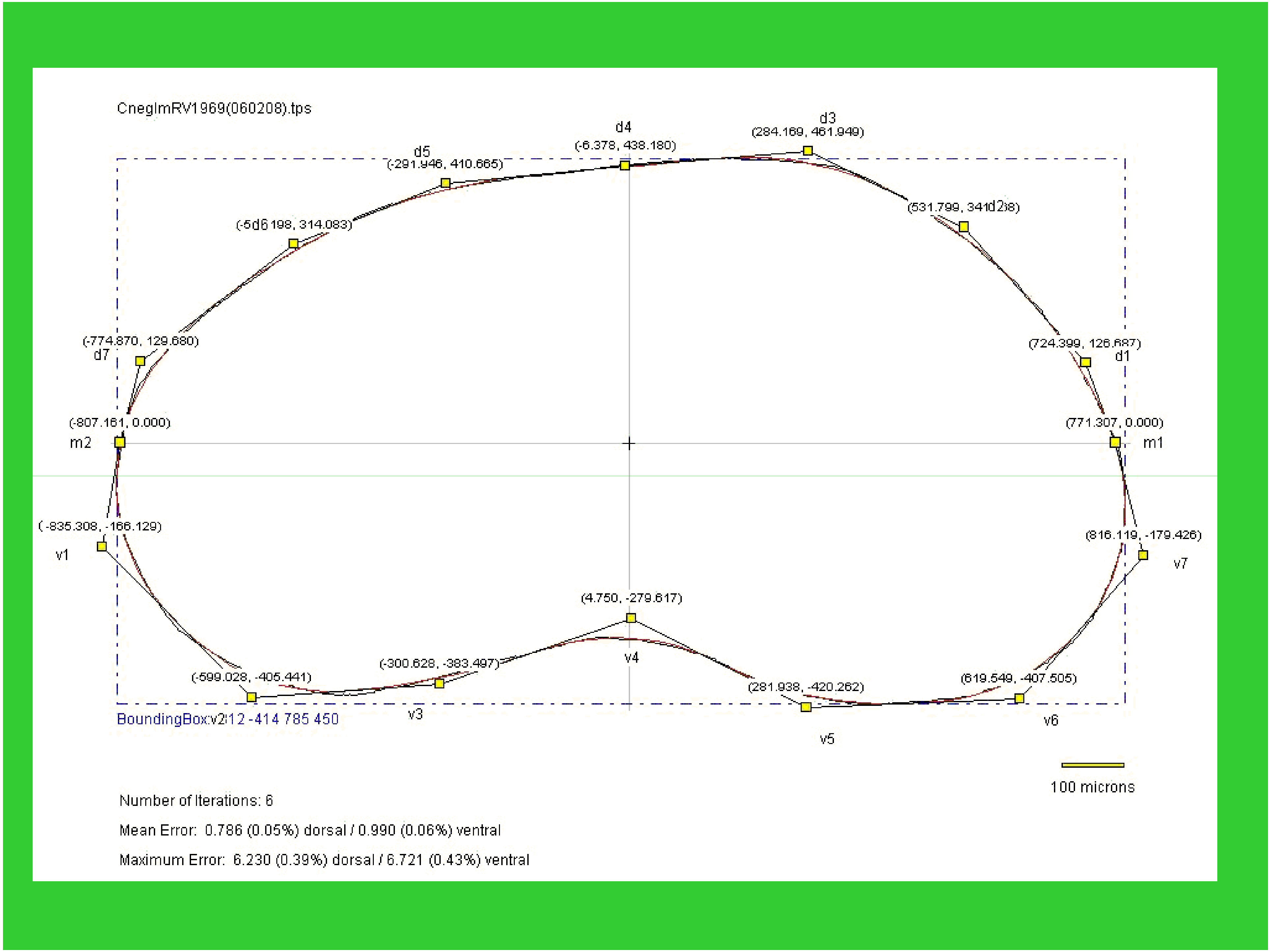
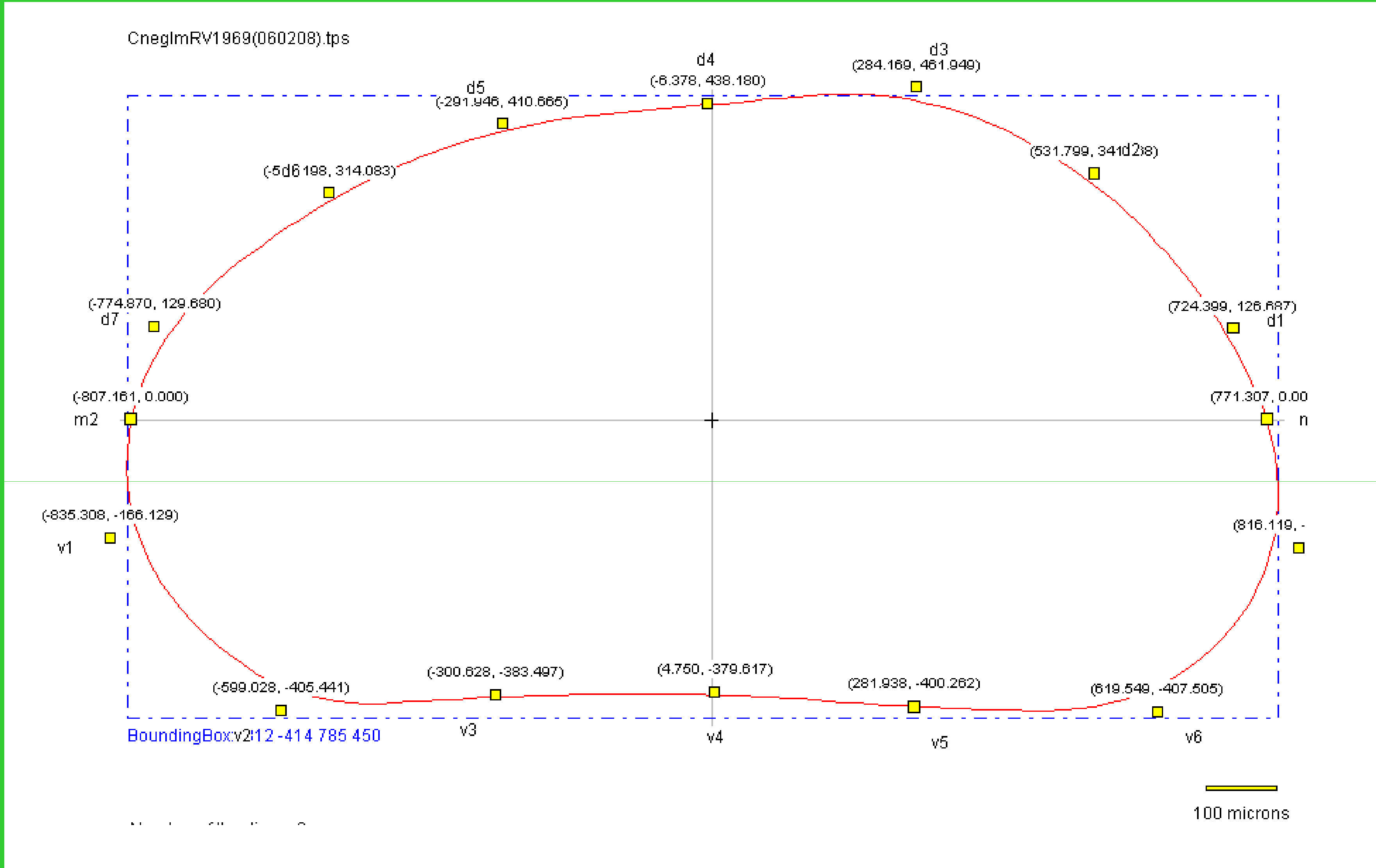
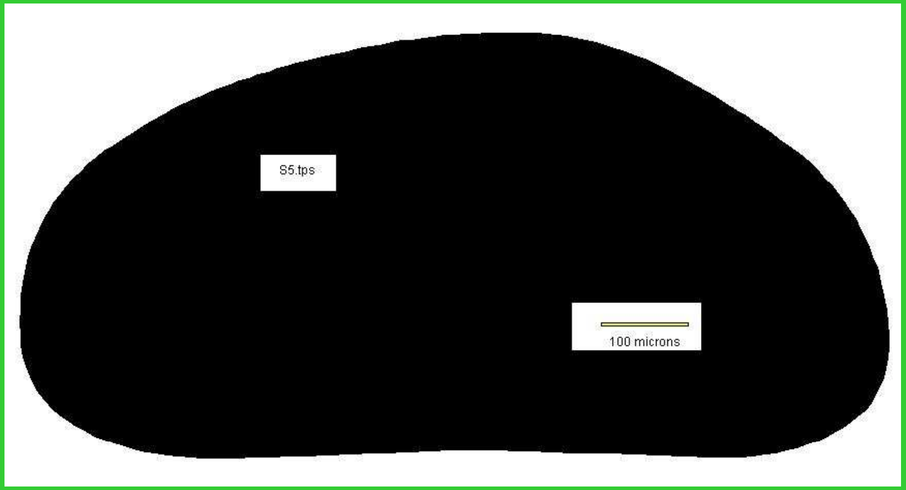

CneglmRV1969(060208).tps
100 microns

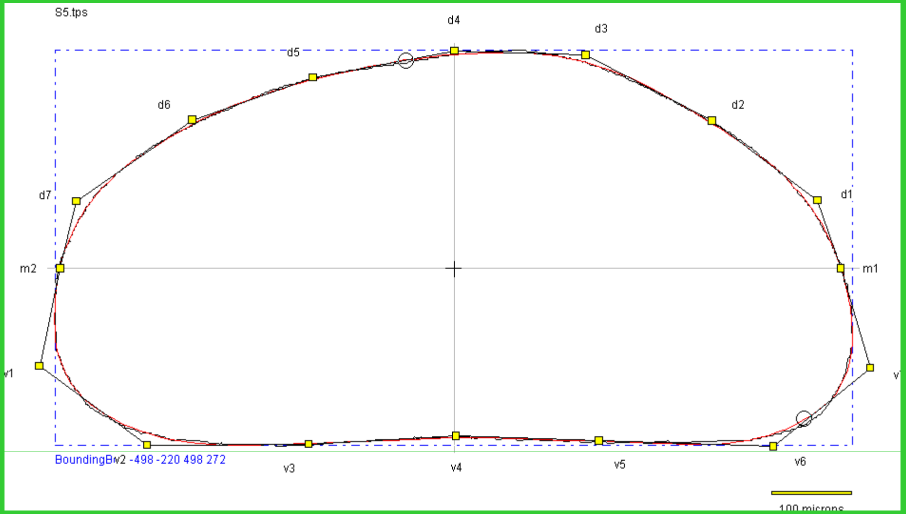

CneglmRV1969(060208).tps
d4
(-6.378, 438.180)
d3
(284.169, 461.949)
d5
(-291.946, 410.665)
(-5d6 198, 314.083)
(531.799, 341d2)8)
(-774.870, 129.680)
d7
(724.399, 126.687)
d1
(-807.161, 0.000)
m2
(771.307, 0.000)
m1
(-835.308, -166.129)
v1
(816.119, -179.426)
v7
(4.750, -279.617)
v4
(-300.628, -383.497)
(-599.028, -405.441)
v3
(281.938, -420.262)
v5
(619.549, -407.505)
v6
BoundingBox:v2!12 -414 785 450
100 microns
Number of Iterations: 6
Mean Error: 0.786 (0.05%) dorsal / 0.990 (0.06%) ventral
Maximum Error: 6.230 (0.39%) dorsal / 6.721 (0.43%) ventral

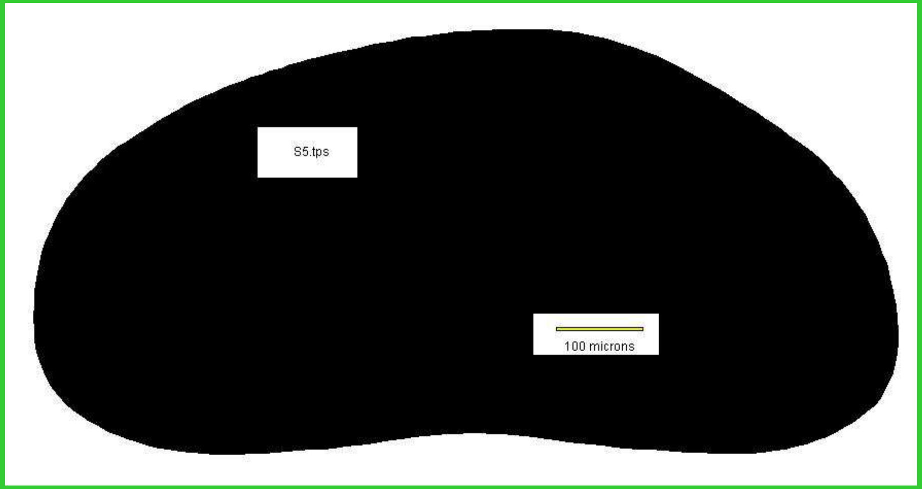


CneglmRV1969(060208).tps
d4
(-6.378, 438.180)
d3
(284.169, 461.949)
d5
(-291.946, 410.665)
(-5d6198, 314.083)
(531.799, 341d2i8)
(-774.870, 129.680)
d7
(724.399, 126.687)
d1
(-807.161, 0.000)
m2
(771.307, 0.00
n
(-835.308, -166.129)
v1
(816.119, -
(-599.028, -405.441)
(-300.628, -383.497)
(4.750, -379.617)
(281.938, -400.262)
(619.549, -407.505)
BoundingBox:v2l12 -414 785 450
v3
v4
v5
v6
100 microns

Modelling of the shape of a male valve through deformation of a female valve of *Cr. racovitzai* at the CP – v4. (A-male's valve of. Cr. vavrai from Namiotko et al., 2005b)







Conclusion

(1) The algorithms used in the computer programme „Morphomatica“ 1.6 are effective tools for reconstruction of the affinites and/or differences between the valve morphology of different populations of Ostracoda as well as for the reconstruction of their evolutionary trajectories both developmental and historical one.

(2) Morphologic traits of valves as compared to those of the limbs, when studied with both classic and geometric morphometrics are able to offer rich information for evolutionary systematics and for ecology.

EPILOGUE

For the harmony of the world is made manifest in Form and Number, and the hart and soul and all the poetry of Natural Philosophy are embodied in the concept of mathematical beauty.

D'Arcy Wentworth THOMPSON

(On Growth and Form, 1917)

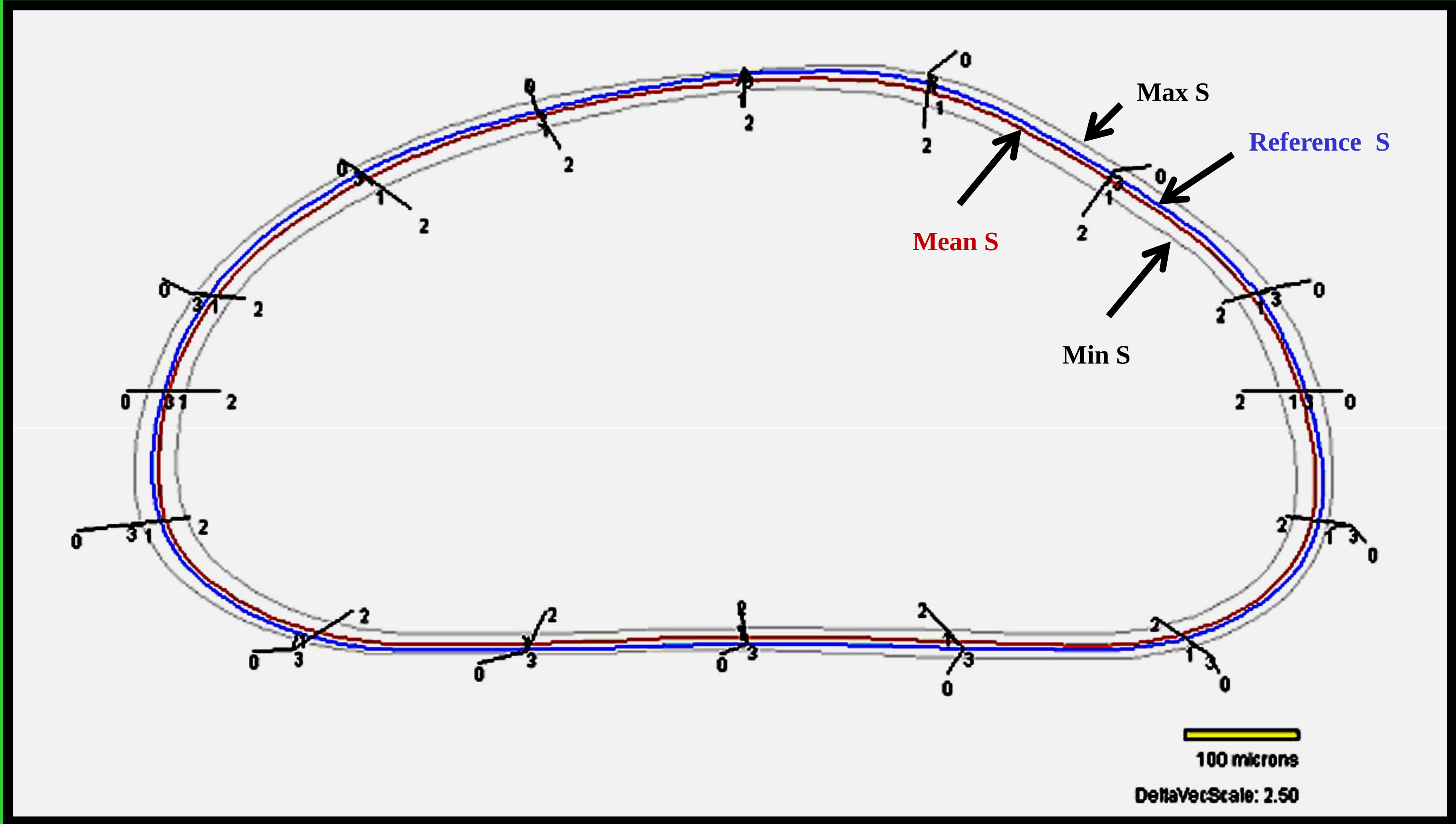
Acknowledgements

This study was supported by the Austrian Science Fund, FWF, (Project P. 17738-B03 to D.L.D.)

Active communication with our colleagues, Prof. Dr. J. Linhart, Dr. A. Baltanás, Prof. F. Österreicher, Mag. W. Brauneis, Mag. K. Minati, Dr. L. Picot, Prof. W. Piller, Prof. K. Martens, Dr. M. Gross, Dr. R. Clarke was very useful.

The technical help of A. Stracke, J. Knoblechner, Mag. M. Pichler and Mag. A. Danielopol, in Mondsee, is also specially acknowledged.

Protocol of valve outlines representation for a given species/population (reference shape should be the holotype)



Quantitative evaluation of Disparity

WD = Within Population Disparity

$$\mathbf{WD_1} = (\sum d(\mathbf{P_1}) / \mathbf{n_1}) \; ; \; \mathbf{WD_2} = (\sum d(\mathbf{P_2}) / \mathbf{n_2})$$

BD = Between Populations Disparity

$$\mathbf{BD} = \sum d(\mathbf{P_1},\mathbf{P_2}) / \mathbf{n_1} * \mathbf{n_2}$$

Linhart‘s Disparity Index

$$\mathbf{LDI} = (\sum d(\mathbf{P_1},\mathbf{P_2}) / \mathbf{n_1} * \mathbf{n_2}) / (\sum d(\mathbf{P_1}) / \mathbf{n_1}^2) + (\sum d(\mathbf{P_2}) / \mathbf{n_2}^2)$$

Finnal values are expressed as Ln of LDI scaled from 0 (complete similarity within and between populations) to ∞ dissimilarity .

See for descrition of the LDI algorithm, Neubauer‘s master thesis at http://palstrat.uni-graz.at/morphomatica/